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LABORATORY REPORT

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Technical Report No. 36 July 1967

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA

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A report on research sponsored by the

Office of Naval Research
Contract Nonr 220(50)

(Submitted to Physics Letters for publication)

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ABSTRACT

It is shown that temporal echoes can occur in collision-free plasmas at various times for different wave number combinations and that spatial echoes can occur at various positions for different frequency combinations.

A plasma wave echo in collision-free plasmas has been predicted from a perturbation solution of the Vlasov equation[1]. Here we demonstrate the existence of higher order temporal as well as spatial echoes, based on a simple nonperturbation method which gives results to all orders. The self-consistent field of the plasma is, however, neglected.

We consider a plasma of charged particles subjected to two external electric field pulses with different wave numbers at $t = 0, \tau$, i.e. $qE_x/M = v_1\ \delta(t) \cos k_1 x + v_2\ \delta(t-\tau) \cos k_2 x$. The position of a charged particle whose initial position and velocity is (x_o, u_o) in such a field is readily found to be

$$\begin{aligned} x(t,x_o,u_o) &= x_o + t(u_o + v_1 \cos[k_1 x_o]) && 0 \leq t \leq \tau \\ &= x(\tau,x_o,u_o) + v_2(t-\tau) \cos[k_2 x(\tau,x_o,u_o)] && \tau \leq t < \infty \end{aligned} \tag{1}$$

The spatial Fourier components of the charge density may be obtained by integrating over the initial (Maxwellian) distribution $F_o(x_o,u_o)$, $\rho_k(t) = q \iint F_o(x_o,u_o) \exp[-ikx(t,x_o,u_o)]\, dx_o\, du_o$. Expanding the exponential using well known Bessel function identities[2] and performing the integrations, we find that $\rho_k(t)$ vanishes unless

$$k = -mk_1 + nk_2 \equiv k_{mn} \tag{2}$$

whereupon

$$\rho_k(t) = n_o q\, A_{mn}(t)\, g_{mn}(t - \tau_{mn}) \tag{3}$$

with

$$\tau_{mn} = (nk_2/k_{mn})\,\tau \tag{4}$$

$$g_{mn}(t) = (1/m!)(k_{mn}\bar{v}t/2)^m\, e^{-(k_{mn}\bar{v}t/2)^2} \tag{5}$$

$$A_{mn}(t) = (-i)^{n-m}\,\frac{J_m[k_{mn}v_1(t-\tau_{mn})]}{[k_{mn}\bar{v}(t-\tau_{mn})/2]^m/m!}\; J_n[k_{mn}v_2(t-\tau)] \tag{6}$$

n_o is the average density of charged particles and $\bar{v} = \sqrt{2\kappa T/M}$ is their r.m.s. thermal speed. When v_1 and v_2 are small compared with $\bar{v}$, $A_{mn}(t)$ is slowly varying compared to $g_{mn}(t)$. Furthermore, only $A_{mn}(t)$ depends on the pulse amplitudes v_1 and v_2 . In this limit $A_{mn}(\tau_{mn})$ gives the amplitude of the pulse and $g_{mn}(t)$ gives its shape. Thus we see that, provided $\tau_{mn} > \tau$, each spatial Fourier component with combination wave number given by (2) exhibits a temporal echo at time τ_{mn} which is determined by the separation of the two applied pulses and the ratio of their wave numbers. For weak pulses (v_1 and v_2 small) the amplitude reduces to

$$A_{mn} = (-i)^{n-m}\left(\frac{v_1}{\bar{v}}\right)^m \left(\frac{v_2}{\bar{v}}\right)^n \frac{1}{n!}\left(\frac{mk_1\bar{v}\,\tau}{2}\right)^n \tag{7}$$

which exhibits the expected power law dependence on the pulse amplitudes v_1 and v_2 and the pulse separation τ .

A similar analysis has been made for spatial echoes, which may be more important experimentally. When spatially localized electric fields of differing frequencies ω_1 and ω_2 are applied at $z = 0$ and $z = \ell$ respectively, in the plasma, $E_x = V_1\,\delta(x)\cos\omega_1 t + V_2\,\delta(x-\ell)\cos\omega_2 t$. The temporal Fourier components of the electric current density are found to vanish unless

$$\omega = - m\omega_1 + n\omega_2 \equiv \omega_{mn} \tag{8}$$

and that each of these exhibits a localized maximum at the position

$$x = (n\omega_2/\omega_{mn})\ell \equiv \ell_{mn} \tag{9}$$

provided that $\ell_{mn} > \ell$. We also find for the amplitude

$$A_{mn}(\ell_{mn}) \sim \left[\frac{m\omega_1 \ell}{2\bar{v}} \frac{eV_2}{M\bar{v}^2}\right]^n \left[\frac{n\omega_2 \ell}{2\bar{v}} \frac{eV_1}{M\bar{v}^2}\right]^m \tag{10}$$

when the dimensionless quantities in the brackets are small. (8), (9), and (10) are the spatial analogs of (2), (4) and (6), respectively. Thus each temporal Fourier component of the electric current with frequencies given by (8) exhibits a spatial echo at a position which is determined by (9).

In obtaining these results the self-consistent fields of the plasma have been neglected. This approximation should be valid as long as all wave numbers for which ρ_k/ρ_o is important are large (all frequencies for which i_k/i_o is important are large). When these conditions are not satisfied we expect a substantial modification in the echo amplitudes and shapes. However, we still expect (4) [or (9)] to correctly give the time [or position] of the echoes of the appropriate wave number (2) [or frequency (8)]. In addition, for small amplitudes we still expect the power law dependence on v_1, v_2 and τ [or V_1, V_2 and ℓ] suggested by (6) [or (10)] to hold.

The author is indebted to T. M. O'Neil and J. H. Malmberg for helpful discussions and to the Office of Naval Research for its sponsorship of this research.

References

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2. W. Magnus, F. Oberhettinger, Special Functions of Mathematical Physics, Chelsea 1949., New York, see p. 18 and p. 20.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) California Institute of Technology Pasadena, California 91109	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

ECHOES IN COLLISION-FREE PLASMAS

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Scientific Interim

5. AUTHOR(S) (First name, middle initial, last name)

Roy W. Gould

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July 1967	6	2

8a. CONTRACT OR GRANT NO. Nonr 220(50) b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Tech. Report No. 36 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Electronics Division 427 Office of Naval Research Washington, D. C.

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DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Plasma waves						
Echo						
Collisionless damping						